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Onset of gravitactic bioconvection in a square anisotropic porous medium with arbitrary orientation of the principal axes

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Abstract

This paper examines the onset of negative gravitactic bioconvection in a porous square cavity. The porous medium is assumed to be anisotropic in permeability with its principal axes arbitrarily oriented with respect to the gravity vector, and contains a concentration of microorganisms which are assumed to be swimming upward against gravity (negative gravitaxis). The critical Rayleigh number Ra_c at marginal stability is calculated as a function of the microorganism swimming speed V_c, of the anisotropy ratio R, and of the orientation angle θ.

Keywords: Bioconvection; Negative gravitaxis; Gravitactic microorganisms; Anisotropic porous medium

1. Introduction

Contrary to Bénard convection which has been widely studied over the past century, bioconvection [1], which is a natural phenomenon observed in suspensions of collective swimming microorganisms constituted from different species of protozoa, algae, or bacteria, has received growing interest only during the past few decades. In fact, negative gravitactic bioconvection is fundamentally analogous to Bénard convection when the microorganism swimming speed is very low, in other words, when V_c tends to zero [2,3].

A model of bioconvection was developed by Childress et al. [4] for gravitactic microorganisms, based on the Navier–Stokes equation for fluid flow and the diffusion-convection equation for the concentration of the motile microorganisms. They first derived the equilibrium state resulting from upward swimming and the downward diffusion of the motile organisms in a quiescent fluid. They then determined analytically the critical Rayleigh number for the onset of convection as well as the preferred wavenumber and growth rates [4].

Kuznetsov and Jiang [5] and Kuznetsov and Avramenko [6] considered bioconvection of gravitactic and gyrotactic microorganisms in an isotropic porous medium, based on the Pedley et al. model [7] and Whitaker theory [8]. The Darcy equation and the diffusion-convection equation were solved numerically to obtain the flow and concentration fields in a square cavity in terms of the permeability of the isotropic porous medium. Other different aspects of gyrotactic and oxytactic bioconvection in porous media are referred to [9–11],

A linear stability analysis of gravitactic bioconvection in an isotropic porous cavity was completed by Nguyen et al. [2,12]. By applying a re-normalization approach on a chosen length scale, they deduced a universal curve of linear stability of gravitactic bioconvection for the case of high swimming speeds in an isotropic porous medium [2,12].

The present study investigates the onset of negative gravitactic bioconvection in porous media with the effects of permeability anisotropy. The effects of different orientations of the principal axes of an anisotropic medium with

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Nomenclature

D_c	cell diffusivity, m^2/s
$F = L/H$	shape factor of 2D porous cavity
$\overline{K}$	second-order tensor of permeability
K_1	permeability, m^2
K_2	lateral permeability, m^2
L	length of the porous cavity, m
n	cell concentration, $cells/m^3$
$\bar{n}$	mean cell concentration, $cell/m^3$
n_0	cell concentration at the bottom boundary, $cells/m^3$
n_1	cell concentration at the top side, $cells/m^3$
N	cell dimensionless concentration
$\overline{N}$	mean cell dimensionless concentration
H	height of the porous cavity, m
P^*	dynamic pressure, Pa
P	dimensionless pressure
Ra	Rayleigh number based on the H scale $Ra = gK_2H\beta_b\Delta n/\nu D_c$
Ra^*	Rayleigh number based on the D_c/V_c^* scale $Ra^* = gK_2H\beta_b\bar{n}/\nu D_c$
R	anisotropy ratio $R = K_2/K_1$
$\vec{V}^*$	Darcy velocity, m/s
$\vec{V}_c^*$	cell gravitactic swimming speed, m/s
$\vec{V}$	dimensionless Darcy velocity $\vec{V} = H\vec{V}^*/D_c$
$\vec{V}_c$	cell dimensionless swimming speed $\vec{V}_c = H\vec{V}_c^*/D_c$ or Peclet number
(X, Y, t^*)	Cartesian coordinates, m and time, s
(x, y, t)	dimensionless Cartesian coordinates and dimensionless time $x = X/H$; $y = Y/H$, $t = D_ct^*/H^2$
Greek symbols	
β_b	density variation coefficient of suspension $\beta_b = \vartheta\Delta\rho/\rho_0$
Δn	cell concentration difference between top and bottom ($cell/m^3$) $\Delta n = n_1 - n_0$, $cells/m^3$
μ	dynamic viscosity of fluid (1 $N\ s/m^2$)
$\Delta\rho$	$\Delta\rho = \rho_c - \rho_w$, kg/m^3
ψ^*	stream function, m^2/s
ψ	$\psi = \psi^*/D_c$ dimensionless stream function
ν	kinematic viscosity of suspension, m^2/s
ρ_w	water density, kg/m^3
ρ_c	cell density, kg/m^3
ρ	density of suspension "fluid-cell", kg/m^3
ρ_0	density of suspension at the bottom of the cavity, kg/m^3
θ	oblique angle of the principal axes – orientation angle

respect to the gravity vector are fundamentally important since this situation frequently occurs in real porous media systems. For the study of natural convection in porous media with the effects of permeability anisotropy, we refer to [13–16].

The results herein highlight the first study of bioconvection in an anisotropic porous medium as well as provide insight into several fundamental processes and give direction for our future research involving bioconvection in heterogeneous porous media. The results are also relevant to a number of geophysical and environmental problems because subsurface media, which contains all five major groups of microorganisms (bacteria, actinomycetes, fungi, algae and protozoa) as well as viruses [17], are not ideally isotropic. The necessity for a model of microbial transport in heterogeneous media is underlined by Li et al. [18]. The problem of bioconvection is found in numerous applications such as the study of the effects of toxins on the negative gravitactic behavior of the protozoa *Tetrahymena pyriformis* (Fig. 1) or *Chlamydomonas reinhardtii* [19].

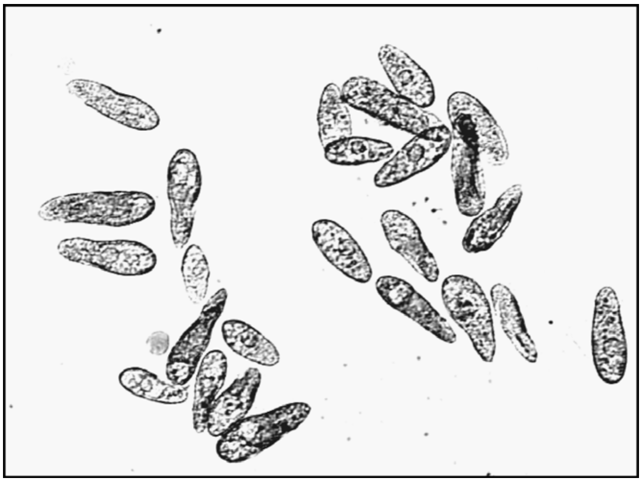

Fig. 1. Axenic culture of gravitactic protozoa *Tetrahymena pyriformis*. (Photo taken at the Institute for Biotechnology and Bioengineering, Centre for Biological Engineering, University of Minho, Portugal by *T.* Nguyen-Quang).

2. Formulation of the problem

For the conceptual model of bioconvection in a porous medium, the governing equations for unsteady flow in a porous medium are obtained by volume averaging the equations of Pedley et al. [7] according to the theory of Whitaker [8] as described in Kuznetsov and Jiang [5]. The replacement of the Laplacian viscous terms with the Darcian terms describing viscous resistance in a porous medium is the basic concept of this approach [20]. The validity of Darcy's law in the porous medium bioconvection model is therefore assumed similar to that in a natural convection model in a layer being heated from below.

The following hypotheses are also assumed in order to ensure that gravitactic behavior is not disturbed: (1) the

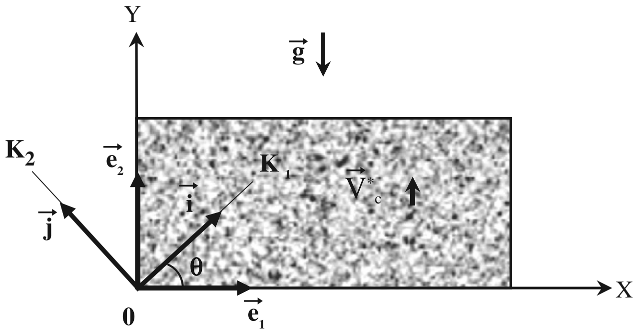

Fig. 2. Physical description of the problem.

porous matrix does not absorb microorganisms, (2) the pore sizes are significantly larger than the microbial cell sizes and the microorganism suspension is dilute; therefore, the change of permeability of the porous matrix due to cell deposition is negligible, (3) the possible local vorticity generated by flow through the pores does not affect the ability of microorganisms to reorient themselves; (4) the mortality and the multiplication of microorganism cells are neglected, in other words, the cells are assumed not to die or grow; the number of cells is therefore constant.

We consider a 2D anisotropic porous layer confined between four impermeable walls with a height H and length L, containing an initial density of gravitactic microorganisms $\bar{n}$, i.e. swimming vertically upwards, with a mean constant speed $\vec{V}_c^*$ as shown in Fig. 2. The Cartesian coordinates (OX, OY) are oriented with the unit vectors $\vec{e}_1, \vec{e}_2$, respectively. We assume that all physical properties of the fluid are constant except the density in the buoyancy term of the Boussinesq approximation. The porous medium is assumed anisotropic with a permeability tensor $\overline{K}$. It is also assumed that the suspension of microorganisms is incompressible.

2.1. Dimensional governing equations

The dimensional governing equations of gravitactic bioconvection in a 2D anisotropic porous medium are described as follows:

$$\nabla \cdot \vec{V}^* = 0 \tag{1}$$

$$-\vec{\nabla}P^* - \frac{\mu}{\overline{K}}\vec{V}^* + \vec{g}\rho = 0 \tag{2}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{V}^*) + \nabla \cdot (n\vec{V}_c^*) = D_c\nabla^2 n \tag{3}$$

$$\rho = \rho_0(1 + \beta(n - n_0)) \tag{4}$$

where $\vec{V}^*$ is the Darcy velocity, $\vec{V}_c^*$ and n are the mean negative gravitactic swimming speed and the concentration of microorganisms, respectively. $\beta = \Delta\rho\vartheta/\rho_0$ is considered as the bioconvective expansion coefficient and ρ_0 is the cell density at the bottom layer of the media. D_c stands for the cell diffusivity.

The parameter $\overline{K}$ in Eq. (2) is a second-order tensor which is defined as follows [21]:

$$\overline{K} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \quad \text{with} \quad \begin{cases} K_{12} = K_{21} = (R-1)\cos\theta\sin\theta \\ K_{11} = R\cos^2\theta + \sin^2\theta \\ K_{22} = R\sin^2\theta + \cos^2\theta \\ R = K_2/K_1 \end{cases} \tag{5}$$

2.2. Initial and boundary conditions

Since the boundaries are impermeable, the normal component of the velocity and the cell flux must be equal to zero.

At $t^* = 0$, we assume that the initial concentration is uniform

$$n(X, Y, t^*) = n(X, Y, 0) = \bar{n} \tag{6}$$

At the impermeable boundaries, the condition of zero-normal fluid velocity requires

$$\begin{cases} X = 0, L: & V_X^* = 0 \\ Y = 0, H: & V_Y^* = 0 \end{cases} \tag{7}$$

while the condition of zero-concentration flux is

$$\vec{J}^* \cdot \vec{e} = [-D_c\nabla n + n(\vec{V}^* + \vec{V}_c^*)] \cdot \vec{e} = 0 \tag{8}$$

$$\begin{cases} X = 0, L: J_X^* = -\partial n/\partial X = 0 \\ Y = 0, H: J_Y^* = nV_c^* - D_c\partial n/\partial Y = 0 \end{cases} \tag{9}$$

where $\vec{e}$ is the unit normal vector to the boundaries.

2.3. Diffusive equilibrium state

For $\vec{V}_c^* = (0, V_c^*)$, the system (1)–(4) under the initial and boundary conditions equations (6)–(9) gives the following steady state solution $\vec{V}^* = 0$ and

$$n = \frac{\bar{n}\left(\frac{V_c^* H}{D_c}\right)}{\exp\left(\frac{V_c^* H}{D_c}\right) - 1}\exp\left(\frac{V_c^*}{D_c}Y\right) \tag{10}$$

which satisfies the conservation of concentration

$$\bar{n} = \frac{1}{LH}\int_0^L \mathrm{d}X \int_0^H n(X, Y, t^*)\mathrm{d}Y \tag{11}$$

Let $V_c = V_c^* H/D_c$ be the dimensionless cell velocity, then Eq. (10) can be re-written as

$$n = \frac{\bar{n}V_c e^{\frac{V_c}{H}Y}}{e^{V_c} - 1} \tag{12}$$

from which we readily deduce the concentrations at the bottom ($Y = 0$) and the top ($Y = H$) of the cavity

$$n_0 = \frac{V_c}{e^{V_c} - 1}\bar{n}; \quad n_1 = \frac{V_c e^{V_c}}{e^{V_c} - 1}\bar{n} \tag{13}$$

and

$$\Delta n = (n_1 - n_0) = \bar{n}\left(\frac{V_c^* H}{D_c}\right) = \bar{n} V_c \tag{14}$$

2.4. Dimensionless form in terms of the stream function ψ

The system of governing equations is *normalized* in terms of the stream function ψ:

$$\begin{cases} \nabla(\overline{K}\nabla\psi) = Ra\frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial t} + \nabla\cdot(N\vec{V}) + \nabla\cdot(N\vec{V}_c) = \nabla^2 N \end{cases} \tag{15}$$

By expanding the permeability tensor, the system of equations becomes:

$$\begin{cases} K_{11}\frac{\partial^2\psi}{\partial x^2} + K_{12}\frac{\partial^2\psi}{\partial x\partial y} + K_{22}\frac{\partial^2\psi}{\partial y^2} = Ra\frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial t} + \nabla\cdot(N\vec{V}) + \nabla\cdot(N\vec{V}_c) = \nabla^2 N \end{cases} \tag{16}$$

with

$$\begin{cases} K_{12} = K_{21} = (R-1)\cos\theta\sin\theta \\ K_{11} = R\cos^2\theta + \sin^2\theta \\ K_{22} = R\sin^2\theta + \cos^2\theta \\ R = K_2/K_1 \end{cases} \tag{17}$$

where

$$\vec{V} = \left(\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}\right) \text{ is the stream function definition.} \tag{18}$$

$$Ra = \frac{gK_2H\beta\Delta n}{\nu D_c} \text{ Rayleigh number based on } K_2 \tag{19}$$

From (14), the Rayleigh number is expressed in terms of $\bar{n}$ and V_c

$$Ra = \frac{gK_2H\beta\bar{n}V_c}{\nu D_c} \tag{20}$$

where K_2 stands for the lateral permeability.

2.5. Dimensionless initial and boundary conditions

$$N = \overline{N} = \frac{\bar{n} - n_0}{\Delta n} = \frac{e^{V_c} - V_c - 1}{(e^{V_c} - 1)V_c} \quad \text{at } t = 0 \tag{21}$$

$$\psi = 0, \quad \partial N/\partial x = 0 \quad \text{at } x = 0, \; F = L/H \tag{22}$$

$$\psi = 0, \quad \partial N/\partial y = V_c N + V_c/(e^{V_c} - 1) \quad \text{at } y = 0, 1 \tag{23}$$

The system is then governed by five parameters: the aspect ratio F of the cavity; the Rayleigh number Ra; the anisotropy ratio R; the orientation angle θ and evidently the swimming speed of the microorganisms V_c.

The aspect ratio F will be held at 1, in order to simplify the discussion and to specifically highlight the effects of anisotropy and orientation of the principal axes. From Eq. (15) and boundary conditions (22) and (23), it can be proved that if $\psi(x,y)$ and $N(x,y)$ are the solutions at Ra, R and θ, then $\psi(F-x,y)$ and $N(F-x,y)$ are the solutions at Ra, R and $(\pi-\theta)$. We thus limit the study to the range of θ from *0* to $\pi/2$.

3. Linear stability analysis

Let the diffusion state "*d*" be perturbed by quantities denoted by "1", i.e.:

$$N(x,y,t) = N_d + N_1; \psi(x,y,t) = \psi_1; \vec{V}(x,y,t) = \vec{V}_1 \tag{24}$$

with $N, \psi, \vec{V}$ being the disturbed state with

$$N_1 \ll N_d = \frac{e^{V_c y} - 1}{(e^{V_c} - 1)} \tag{25}$$

By substituting Eq. (24) in Eq. (16) and neglecting second-order terms of perturbations, we obtain

$$\begin{cases} K_{11}\frac{\partial^2\psi_1}{\partial x^2} + K_{12}\frac{\partial^2\psi_1}{\partial x\partial y} + K_{22}\frac{\partial^2\psi_1}{\partial y^2} = Ra\frac{\partial N_1}{\partial x} \\ \frac{\partial N_1}{\partial t} + \nabla.[N_d\vec{V}_1 + N_1\vec{V}_c] = \nabla^2 N_1 \end{cases} \tag{26}$$

or

$$\begin{cases} K_{11}\frac{\partial^2\psi_1}{\partial x^2} + K_{12}\frac{\partial^2\psi_1}{\partial x\partial y} + K_{22}\frac{\partial^2\psi_1}{\partial y^2} = Ra\frac{\partial N_1}{\partial x} \\ \frac{\partial N_1}{\partial t} - G(y)\frac{\partial\psi_1}{\partial x} + V_c\frac{\partial N_1}{\partial y} = \nabla^2 N_1 \end{cases} \tag{27}$$

$$\text{with } G(y) = \frac{V_c e^{V_c y}}{(e^{V_c} - 1)} \tag{28}$$

and the required boundary conditions for (27) are

$$\psi_1 = \frac{\partial N_1}{\partial x} = 0 \quad \text{at} \quad x = 0, F \tag{29}$$

$$\psi_1 = \frac{\partial N_1}{\partial y} - V_c N_1 = 0 \quad \text{at } y = 0, 1 \tag{30}$$

The linear system (27) determines the initial evolution of perturbations and the criterion for the onset of bioconvection. It is noted that when the cell velocity $V_c \to 0$, $G(y) \to 1$, and the above equations (27) reduce to the well-known equations governing the fixed-flux problem in an anisotropic porous cavity studied by Mamou et al. [15].

When the angle θ approaches zero, $K_1 = K_2 (R = 1)$, we obtain the pure gravitactic bioconvection problem in an isotropic porous medium treated by Nguyen-Quang et al. [2] and Nguyen-Quang [3]. Our newly-developed numerical code was successfully benchmarked using the results from the limiting cases of pure gravitactic bioconvection and thermal convection in an anisotropic porous medium heated by heat flux.

4. Numerical solution approach

If the orientation angle θ is non-zero, the solution of Eqs. (27)–(30) is not limited to a single Fourier component in either the vertical or horizontal directions; an analytical process could therefore be tedious and offers no advantages. Hence, the finite-difference numerical method is used for the linear stability analysis. The discretized equations were derived by a central-differencing technique. By arranging the discretized variables ψ_1 and N_1 in two 1D vectors, the system (27) may be expressed under the form:

$$\begin{cases} A\Psi_1 - RaBT_1 = 0 \\ C\Psi_1 - DT_1 = 0 \end{cases} \tag{31}$$

where A, B, C, D are $M \times M$ matrices whose components are functions of the anisotropic permeability R, the orientation angle of the principal axes θ, and the microorganism swimming speed V_c. $M = (M_x - 2)(M_y - 2)$ with M_x and M_y being grid numbers in the horizontal and vertical directions, respectively.

The system (31) is finally transformed into the eigenvalue problem:

$$(E - \lambda I)\Psi_1 = 0 \tag{32}$$

where I is the unit matrix and $E = CA^{-1}BD^{-1}$. In other words, if $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_n$ are the eigenvalues of Eq. (32), then $Ra_c = \frac{1}{\lambda_1}$ is the critical Rayleigh number, and the eigenvector $\psi 1$ corresponding to λ_1 represents the flow pattern at the onset of bioconvection.

The precision of the critical Rayleigh number depends on the grid resolution M_x and M_y. The choice of grid size was adopted after mesh refinement tests. We accepted the grid for which the variation in the solution was <5%. Hence, the results presented in the following sections were obtained with grids of 21 × 21 nodes ($\Delta x = \Delta y = 0.032$) for a square enclosure ($F = 1$).

By letting the swimming velocity tend to zero, i.e. the microorganisms become inanimate (immobile), our numerical code was verified to converge to the thermoconvection problem governed by the fixed-flux condition in an anisotropic porous cavity studied by Mamou et al. [15] (see Table 1). By letting the anisotropic angle θ approach zero and letting $K_1 = K_2 (R = 1)$, we obtained the pure gravitactic bioconvection results studied by [2,3,12] (see Table 2).

The following choice of two representative values of dimensionless microorganism velocities ($V_c = 1$ and 5) for the present linear stability analysis is explained by the fact that a universal stability curve is obtained by a normalization based on the length scale D_c/V_c^* instead of the length scale H in the case of swimming speeds higher than 1 [3,12]. For this case, the renormalized Rayleigh number Ra_c^* is used instead of the usual critical Rayleigh number Ra_c according to the definition of (20). Nguyen-Quang et al. [3,12] show that

$$Ra_c^* = Ra_c / V_c \tag{33}$$

With this formula, we can see that in the case of $V_c = 1$, $Ra_c^* = Ra_c$. To facilitate the following discussion, we will refer only to Ra_c^* for both cases $V_c = 1$ and 5.

Table 1
Validation of the code for the case $V_c = 0$

$\theta = 0$, square cavity	Ra_c, Mamou et al. [15]	Ra_c, Kimura et al. [22]	Ra_c of the present study by letting $V_c = 0$
$R = 10^{-4}$	π^2		9.9497
$R = 10^2$	$10.03R = 1003$		1053
$R = 1$	22.9	22.9	22.92

Table 2
Validation of the code for the case $\theta = 0$ and $K_1 = K_2$ ($R = 1$)

V_c	Ra_c^* according to Kuznetsov and Jiang converted in our definition [5]	Ra_c^* by linear stability analysis Nguyen-Quang et al. [2,12]	Ra_c^* in the present study by letting $\theta = 0$ and $K_1 = K_2$ ($R = 1$)
5		10.2	10.22
10	9.81	10.33	10.315

5. Results and discussion

The critical Rayleigh number Ra_c^* for the onset of motion in a square cavity as a function of R in the case of $\theta = 0$ is presented in Fig. 3 for $V_c = 1$ and 5. For a small value of R, this value tends to 8.96 ($V_c = 1$) and to 2.62 ($V_c = 5$). The critical Rayleigh number Ra_c^* is observed to linearly increase with R. Physically, an increase in R may be interpreted as a decrease in the permeability K_1, since Ra_c^* is based on K_2. From Darcy's law, the velocity is proportional to permeability; therefore any increase in permeability is likely to hasten the onset of bioconvection. When R is large enough, Ra_c^* tends asymptotically towards the value 10.63R (for $V_c = 1$) and $2.522R$ (for $V_c = 5$).

The effects of the angle θ on the critical Rayleigh number are shown in Fig. 4a (for $V_c = 1$) and Fig. 4b (for $V_c = 5$). In the case of a low swimming speed ($V_c = 1$), we observe that Ra_c^* reaches *a minimum value*. This tendency is less clear when R tends to 1 and disappears at $R = 1$ (Fig. 4a). In the case of a high swimming speed ($V_c = 5$), Ra_c^* is observed to reach *a maximum value*. The angle for the maximum value of Ra_c^* depends on R (Fig. 4b).

The following example illustrates the case of $R = 0.001$ for both cases of $V_c = 1$ and 5 (Fig. 5). For $V_c = 1, Ra_c^*$ reaches the minimum value at $\theta = 47°$ ($Ra_c^* = 8.135$), and for $V_c = 5, Ra_c^*$ reaches the maximum value at $\theta = 26°$ ($Ra_c^* = 3.2055$) (Fig. 5).

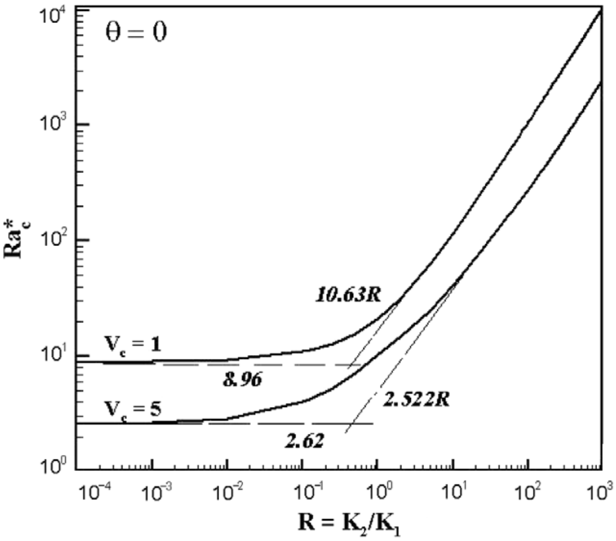

Fig. 3. Effects of the anisotropy ratio R on Ra_c^* for $V_c = 1$ and 5.

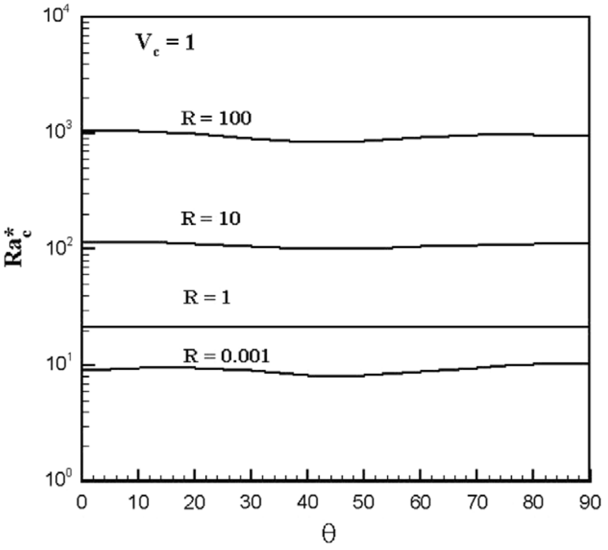

Fig. 4a. Effects of the orientation angle θ on Ra_c^* for $V_c = 1$ and for various anisotropy ratios R.

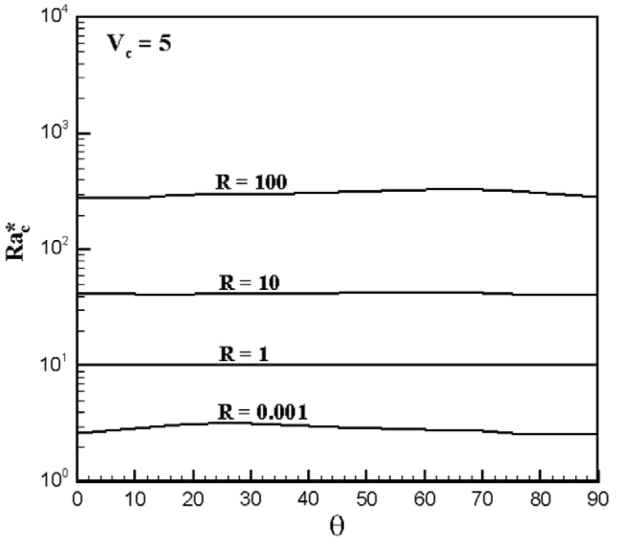

Fig. 4b. Effects of the orientation angle θ on Ra_c^* for $V_c = 5$ and for various anisotropy ratios R.

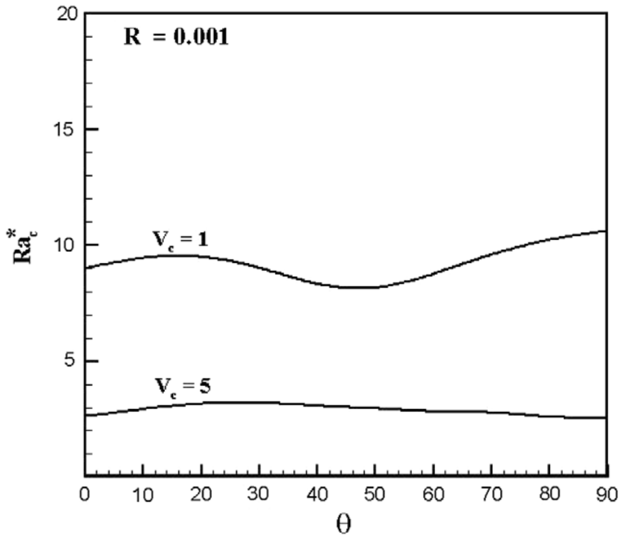

Fig. 5. Effects of the orientation angle θ on Ra_c^* for $V_c = 1$ and 5 and for $R = 0.001$.

The morphology of bioconvection cells is discussed next. The isolines (streamlines and iso-concentrations) of the flow patterns for the onset of gravitactic bioconvection are depicted in Figs. 6a and 6b ($V_c = 1$) and in Figs. 7a and 7b ($V_c = 5$).

The cell pattern in the case of $V_c = 1$ is shown in Figs. 6a and 6b. In Fig. 6a, the flow pattern at the onset of bioconvection is not affected by $R < 320$ when $\theta = 0$. The flow therefore remains unicellular. When the value of R is equal to or greater than 320, the flow becomes multicellular (Fig. 6a).

In the case of $V_c = 5$ when $\theta = 0$, the flow remains unicellular with $R < 1.365$. The flow becomes bicellular when R is above 1.365 and becomes multicellular as R increases (Fig. 7a).

The multicellular structure also depends on the orientation angle θ when $R \neq 1$. Figs. 6b and 7b show that with $R = 0.001$, flow is unicellular when θ is <82.5° (for $V_c = 1$) or <30° (for $V_c = 5$). In other words, the higher the swimming speed V_c, the narrower the range of oblique angle θ for one cell flow pattern, i.e., the flow remains uni-

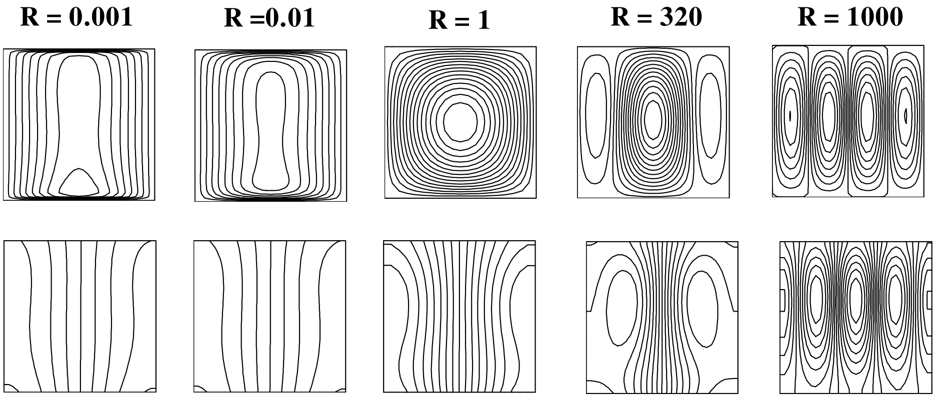

Fig. 6a. Streamlines (*above*) and isoconcentration lines (*below*) at the onset of bioconvection for different R in the case of $V_c = 1$, $\theta = 0$.

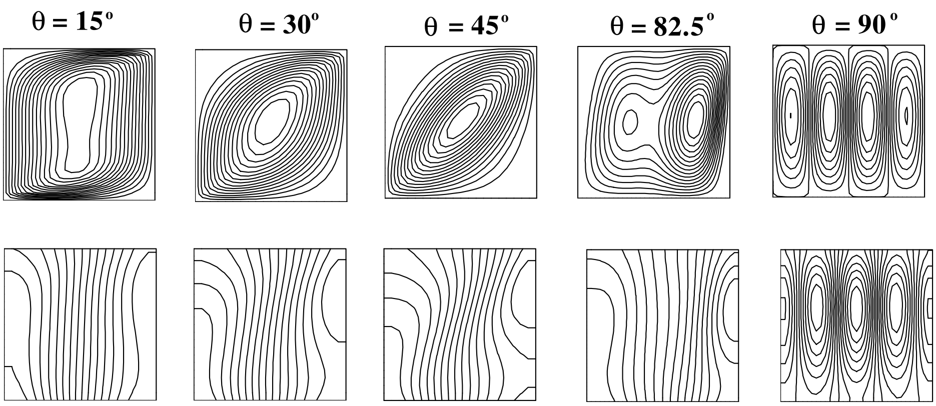

Fig. 6b. Streamlines (*above*) and isoconcentration lines (*below*) at the onset of bioconvection for different θ in the case of $V_c = 1$, $R = 0.001$.

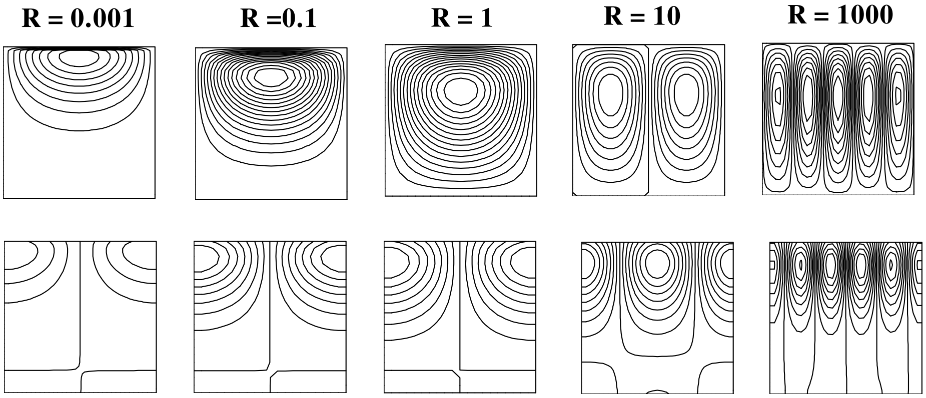

Fig. 7a. Streamlines (*above*) and isoconcentration lines (*below*) at the onset of bioconvection for different R in the case of $V_c = 5$, $\theta = 0$.

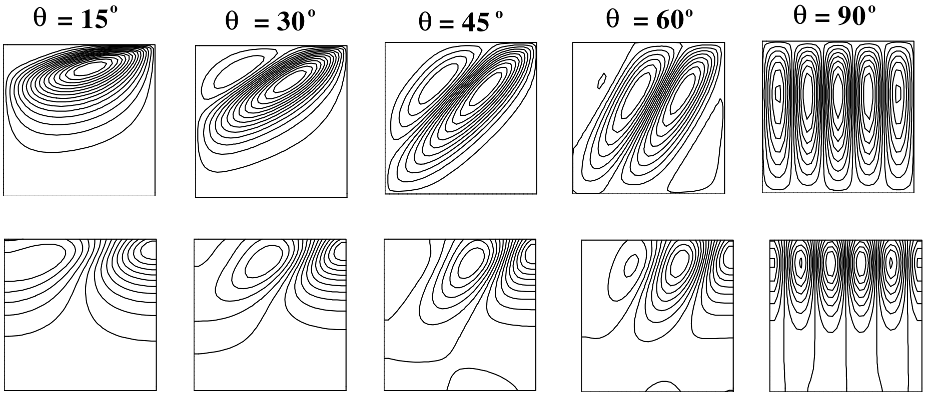

Fig. 7b. Streamlines (*above*) and isoconcentration lines (*below*) at the onset of bioconvection for different θ in the case of $V_c = 5$, $R = 0.001$.

cellular at the angle $\theta \in [0°, 82.5°]$ for $V_c = 1$ and $\theta \in [0°, 30°]$ for $V_c = 5$.

The flow pattern in all cases of V_c is independent of θ when $R = 1$.

6. Conclusions

From the above results, it may be concluded that:

- The swimming speed of gravitactic microorganisms affects the critical stability of incipient bioconvection. The critical Rayleigh number Ra_c^* in the case of swimming speed $V_c = 5$ is less than the critical number in the case of $V_c = 1$.
- Provided that $\theta = 0°$ or $\theta = 90°$, the unicellular morphology at the onset of bioconvection is affected by a change in permeability when $V_c = 1$, and is more significantly affected when $V_c = 5$.
- A change either in permeability or angle of anisotropy significantly influences the flow structure, provided the angle θ is neither 0° nor 90°, especially when the swimming speed is high. The flow is enhanced in the diagonal region.

- From our study, when $R = 0.001$, the unicellular morphology is the preferred mode in the case of $V_c = 1$ in the range of $\theta \in [0°, 82.5°]$ and in the range of $\theta \in [0°, 30°]$ for $V_c = 5$.
- When $\theta = 0$, unicellular flow exists where $R < 320$ for $V_c = 1$ or $R < 1.365$ for $V_c = 5$.

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